# *Modelling randomly occurring events*
## *Risk Analysis Tip (a sample ModelAssist topic)*

A very common problem in risk analysis modelling is how to include events that occur randomly in time, space or any other continuum. For example:

- A fire, earthquake, tsunami, flood, volcanic eruption, disease outbreak or other disaster;
- The failure of some machinery, electronic devise, etc;
- A sudden shock to the stock market;
- Deaths, murders, illnesses, car accidents, plane crashes, fuel spill, toxic release…

The list is endless. In this tip we'll assume that you are modelling events occurring randomly in time, but the principles discussed here have a much greater range of applications (M0462).

In risk analysis, we usually want answers to the following questions:

- How long will it be before an event occurs?
- How many times will an event occur in a certain period?
- What is the probability that there will be at least one event in a certain period?

**The Poisson process**
The Poisson process (M0118) is a conceptual model of a type of random behaviour that gives us simple tools to answer these questions. It is based on two simple assumptions, which the risk analyst must first be comfortable with:

***1. The probability of an event occurring at any moment is constant***
Meaning that the probability does not change with time.

***2. Events occur independently***
Meaning that the occurrence of an event does not affect the chances of the next event occurring.

In fact, these assumptions can often be relaxed by adapting your model (M0414) and being careful about the period over which one is modelling. For example, it can be used where there are seasonal variations (e.g. more events occur in the summer, or in a rush-hour), where there are regional variations (e.g. some states, retail outlets, products have more risk than others), can be modified (M0387) where the risk changes with time, etc.

**Using the Poisson process**

Siméon Denis Poisson developed the probability mathematics that gives the following simple equations:

***Notation:***

t is the amount of time we are considering;
$\alpha$ = the number of events that will occur in time t; and
$\lambda$ is the number of events expected to occur in time t.

***Equations*** (M0462)**:**

The number of events $\alpha$ that will occur in time t = Poisson($\lambda$*t)
The time t to wait until observing $\alpha$ events = Gamma(0, 1/$\lambda$, $\alpha$)
  (The special case of time until one event occurs: = Exponential($\lambda$) )
The probability of no event occurring in time t = EXP(-$\lambda$*t)
  (So the probability of at least one event occurring in time t = 1-EXP(-$\lambda$*t) )

***Example 1:***

Historic records show that earthquakes measuring 8 or above on the Richter scale have occurred in your city at the rate of 0.07 per year. Assuming these large earthquakes occur independently:

How many will there be in the next 3 years = Poisson(0.07*3) = Poisson(0.21)
What is the probability there will be at least one earthquake in the next half year = 1-EXP(-0.07*0.5) = 3.4%
How long will it be until the next earthquake = Exponential(0.07)
  *Curiously, this is not affected by how long it's been since the last earthquake*

***Example 2:***

Your company makes loans. It now has 20,000 customers. Historically, 2.3% of customers default on their loans. How many defaults will there be in the next year if this default rate still applies?

Number of defaults = Poisson(20000*0.023*1) = Poisson(460)

ModelAssist provides many other examples.

**Test your understanding!**

If you're modelling random events with the Poisson process, why not test your knowledge with an interactive ModelAssist quiz  (M0017) – one of many that ModelAssist offers on important topics?